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## HERA Constraint on Warped Quantum Gravity

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### Abstract

We study recent data on deep inelastic  $e^+p$  scattering at HERA to constrain the parameters of a Randall-Sundrum-type scenario of quantum gravity with a small extra dimension and a non-factorable geometry.

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Theories with extra dimensions which predict observable consequences at the current high energy accelerators have lately attracted a great deal of interest. Following the original suggestion by Arkani-Hamed, Dimopoulos and Dvali (ADD) [1], there have been numerous studies in the literature [2] which probe consequences of multiple Kaluza-Klein graviton exchange leading to interactions of electroweak strength. The fact that these theories predict quantum gravity effects at TeV scales has been suggested [1] as a solution of the well-known hierarchy problem in the Standard Model (SM). Though novel and interesting, however, the model suggested by ADD, which is based on a factorable  $\mathbf{R}^4 \times (\mathbf{S}^1)^d$  geometry,  $d$  being the number of extra compact dimensions, has the drawback of introducing large compactification radii (amounting to an energy scale as low as  $10^{-13}$  GeV), which effectively introduces a new hierarchy problem. Motivated by this, Randall and Sundrum (RS) [3] have suggested a somewhat different mechanism to solve the hierarchy problem. Instead of writing a factorable metric

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + R_c^2 d\phi_i d\phi_i \quad (1)$$

where the  $\phi_i$  ( $i = 1, d$ ) are extra dimensions compactified with a common radius  $R \sim 1$  mm, they write a *non-factorable* metric

$$ds^2 = e^{-\mathcal{K}R_c\phi} \eta_{\mu\nu} dx^\mu dx^\nu + R_c^2 d\phi^2 \quad (2)$$

involving one extra dimension compactified with a radius  $R_c$ , which is assumed to be marginally greater than the Planck length  $10^{-33}$  cm, and an extra mass scale  $\mathcal{K}$ , which is related to the Planck scale  $M_P^{(5)}$  in the five-dimensional bulk by  $\mathcal{K} [M_P^{(4)}]^2 \simeq [M_P^{(5)}]^3$ . Such a ‘warped’ geometry is

motivated by compactifying the extra dimension on a  $\mathbf{S}^1/\mathbf{Z}_2$  orbifold, with two  $D$ -branes at the orbifold fixed points, *viz.*, one at  $\phi = 0$  ('Planck brane' or 'invisible brane'), and one at  $\phi = \pi$  ('TeV brane' or 'visible brane'). It can then be shown that if we assume matter fields to be confined to these  $D$ -branes, one can solve the Einstein equations to obtain a metric of the above form. The interesting physical consequence of this geometry is that any mass scale  $\mathcal{M}$  on either brane gets scaled by the 'warp factor'  $e^{-\mathcal{K}R_c\phi}$  on either brane. Thus, a mass scale on the Planck brane ( $\phi = 0$ ) will remain unchanged, but any mass scale on the TeV brane will be scaled by a factor  $e^{-\pi\mathcal{K}R_c}$ . If we assume that the Planck scale is the only fundamental mass scale in the theory, all masses on the TeV brane will be scaled to

$$\mathcal{M} \sim e^{-\pi\mathcal{K}R_c} M_P^{(4)} \quad (3)$$

It now requires  $\mathcal{K}R_c \simeq 11 - 12$  to obtain  $\mathcal{M}$  of the order of the electroweak scale, which justifies the name 'TeV brane'. Thus, in this model there is no hierarchy problem, since all the independent mass scales are close to the Planck scale. There still remains a minor problem: that of stabilizing the radius  $R_c$  (which is marginally smaller than the Planck scale) against quantum fluctuations, but this is not so severe as in the model of ADD, where the compactification radius needs to be stabilized over as many as 30 orders of magnitude. A simple extension of the RS construction involving an extra bulk scalar field has been proposed [5] to stabilize  $R_c$  and this predicts light radion excitations with possible collider signatures [6]. However, as these will not contribute to the processes of interest in this letter, this idea will not be discussed further. On the flip side, it is not as simple to embed

the RS construction within the framework of string theories as it is for the ADD case. However, a first attempt has been made [4], and it may be hoped that future work will achieve this highly desirable goal.

Following the ingenious suggestion of a non-factorable geometry, the mass spectrum and couplings of the graviton in the RS model have been worked out, in Refs. [7, 8]. We do not describe the details of this calculation, but refer the reader to the original literature. It is worth noting that there are strong phenomenological constraints on bulk excitations of the SM fields [11]. It suffices here to note that the effective Lagrangian density for graviton interactions on the TeV brane (which we identify with the observable world) has the form [8]

$$\mathcal{L}_{eff}^{RS} = -\frac{1}{\overline{M}_P} h_{\mu\nu}^0(x) T^{\mu\nu}(x) - \frac{e^{\pi K R_c}}{\overline{M}_P} \sum_{n=1}^{\infty} h_{\mu\nu}^n(x) T^{\mu\nu}(x) \quad (4)$$

where  $\overline{M}_P \equiv M_P^{(4)}/\sqrt{8\pi}$  is the reduced Planck mass and the  $h_{\mu\nu}^n(x)$  correspond to the Kaluza-Klein expansion of the massless graviton in five dimensions

$$h_{\mu\nu}(x, \phi) = \sum_{n=0}^{\infty} h_{\mu\nu}^n(x) \frac{\chi^n(\phi)}{\sqrt{R_c}}. \quad (5)$$

Equation (4) tells us that the massless Kaluza-Klein (KK) mode effectively decouples from ordinary matter since its interactions are suppressed by the Planck mass. On the other hand, the massive KK modes couple as the inverse of the Planck mass, scaled by  $e^{-\pi K R_c}$ , which is an electroweak-strength interaction. Feynman rules to the lowest order for these modes, assuming a coupling  $\overline{M}_P^{-1}$  to *all* the modes have been worked out in Refs. [9] and [10] in the context of ADD-like scenarios. All we need to do to get the corresponding

Feynman rules in the RS model is to multiply the couplings by the warp factor  $e^{\pi\mathcal{K}R_c}$  where necessary.

As shown in Ref. [7], the orbifold geometry forces the Fourier coefficients  $\chi^n(\phi)$  to satisfy a Bessel equation, whence it may be shown that they are given by a linear combination of the Bessel and Neumann functions of order 2. The requirement that the first derivative of  $\chi^n(\phi)$  be continuous at the orbifold fixed point  $\phi = \pi$  then requires  $J_1(x_n) = 0$ . Using this, the masses  $M_n$  of the graviton states can be written in terms of the zeros of the Bessel function of order unity as

$$M_n = x_n \mathcal{K} e^{-\pi\mathcal{K}R_c} \equiv x_n m_o \quad (6)$$

where  $m_0$  sets the scale of graviton masses and is essentially a free parameter of the theory. It is also convenient to write

$$\frac{e^{\pi\mathcal{K}R_c}}{M_P} = \frac{c_0}{m_0} \sqrt{8\pi} \quad (7)$$

using (6) and introducing another undetermined parameter  $c_0 \equiv \mathcal{K}/M_P^{(4)}$ . Ref. [8] points out that  $(m_0, c_0)$  may conveniently be taken as the free parameters of the theory, and we follow their prescription in our work.

Though  $c_0$  and  $m_0$  are not precisely known, one can make estimates of their magnitude using theoretical ideas and phenomenological inputs. We note that the RS construction requires  $\mathcal{K}$  to be at least an order of magnitude less than  $M_P^{(4)}$ , because  $\mathcal{K}^{-1}$  sets the scale for the curvature of the fifth dimension, and should therefore be large compared with the Planck length. The latter is necessitated by the requirement that fluctuations in the bulk gravitational field in the vicinity of the  $D$ -branes be small. The range of

interest for  $c_0$  is, therefore, about 0.01 to 0.1, the lower value being determined by naturalness considerations. Regarding  $m_0$ , Eq. (6) tells us that it is reduced from the scale  $\mathcal{K}$  by the factor  $e^{-\pi\mathcal{K}R_c}$ . In the RS construction, one requires  $\mathcal{K}R_c \sim 11\text{--}12$ , which reduces  $m_0$  to the electroweak scale. Hence, we may consider  $m_0$  in the range of a few tens of GeV to a few TeV. Eq. (6) also tells us that the first massive graviton lies at  $M_1 = x_1 m_0 \simeq 3.83 m_0$ . Since no graviton resonances have been seen at LEP-2, running at energies upto 200 GeV, it is clear that we should expect  $m_0 > 52$  GeV.

In this letter we report on a study of graviton effects, within the RS model, on  $e^+p$  deep inelastic scattering (DIS) at HERA. At the leading order, there are two extra Feynman diagrams contributing to  $e^+p \rightarrow e^+ + X$ . One of these involves a  $t$ -channel exchange of a virtual (massive) graviton between the  $e^+$  and a quark; the other involves a  $t$ -channel exchange of a virtual (massive) graviton between the  $e^+$  and a gluon. The first one adds coherently with the corresponding SM diagrams with photon and  $Z$ -boson exchange; the second one has no SM analogue and hence adds incoherently. However, at HERA energies, we do not expect much contribution from the gluon-induced diagram because of the low gluon flux.

The cross-section for the above processes has been calculated for the case of the ADD model in Ref. [12] and can be easily translated to the RS model using the replacement

$$\frac{\lambda}{M_S^4} \longrightarrow \frac{8\pi c_0^2}{m_0^2} \sum_n \frac{1}{|\hat{t}| + M_n^2} \quad (8)$$

We have developed approximate analytic formulae for this sum, using the well-known properties of the zeros of the Bessel function  $J_1(x)$ . These will be

presented elsewhere [13]. Using this, we incorporate the calculated theoretical cross-section into a parton-level Monte Carlo event generator, with two free parameters, *viz.* the graviton mass scale  $m_0$ , and the coupling parameter  $c_0$ . Finally the simulation results are compared with data from the ZEUS Collaboration to constrain the  $(m_0 - c_0)$  plane.

Our numerical studies are founded on the latest results presented [14] by the ZEUS Collaboration, which are based on  $47.7 \text{ pb}^{-1}$  of data collected over the period 1994-1997. The ZEUS Collaboration uses the double-angle (DA) method to determine the DIS variables. In this method, one measures the polar angle  $\theta_e$  of the scattered positron, and reconstructs the polar angle  $\gamma_h$  of the struck quark in the naive parton model using all hadronic clusters which can be identified with a jet having the requisite  $p_T$  balance with the positron. In terms of these observables and the energy  $E_e(E_p)$  of the initial positron (proton) beam, one can reconstruct the standard DIS variables as

$$Q_{DA}^2 = 4E_e^2 \frac{\sin \gamma_h(1 + \cos \theta_e)}{\sin \gamma_h + \sin \theta_e - \sin(\gamma_h + \theta_e)} \quad (9)$$

$$x_{DA} = \frac{E_e}{E_p} \frac{\sin \gamma_h + \sin \theta_e + \sin(\gamma_h + \theta_e)}{\sin \gamma_h + \sin \theta_e - \sin(\gamma_h + \theta_e)} \quad (10)$$

$$y_{DA} = \frac{\sin \theta_e(1 - \cos \gamma_h)}{\sin \gamma_h + \sin \theta_e - \sin(\gamma_h + \theta_e)} \quad (11)$$

Various triggers, acceptances and selection cuts have been used by the ZEUS Collaboration, of which we need to impose only the following in a parton-level analysis:

- If the final state positron has polar angle greater than  $17.2^\circ$ , it must have a total energy greater than 10 GeV.

- If the final state positron has polar angle less than  $17.2^0$ , it must have a transverse momentum greater than 30 GeV.

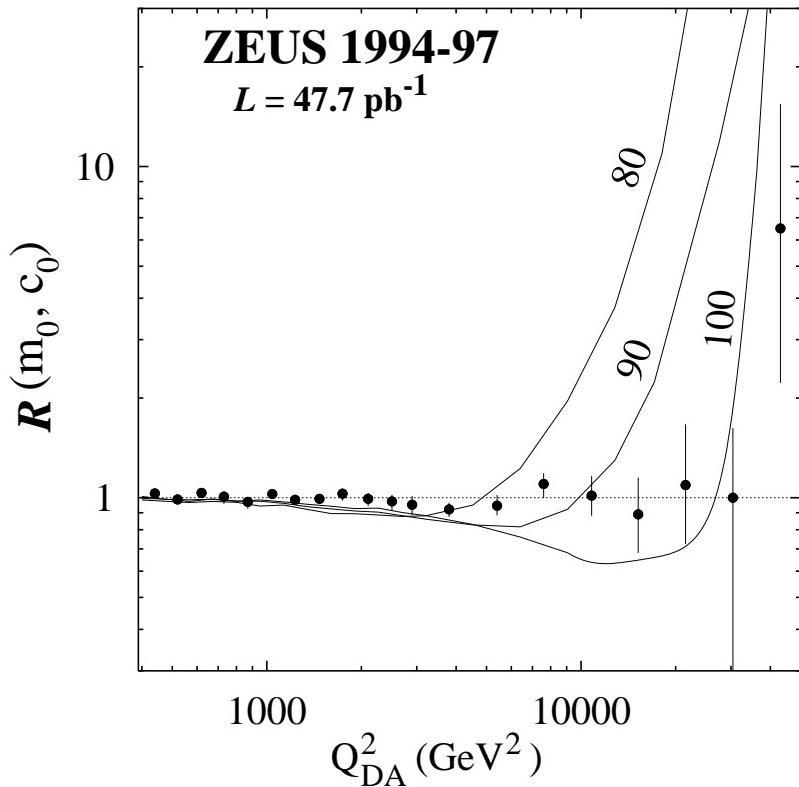
Of the DIS variables listed above, it is well-known that it is the first, namely  $Q_{DA}^2$ , which exhibits maximum sensitivity to most kinds of new physics. The ZEUS Collaboration has presented their data for 20 bins in  $Q_{DA}^2$ , ranging from  $Q_{DA}^2 = 400$  GeV $^2$  to 51200 GeV $^2$ . The Born-level cross-section in each bin, obtained by suitably scaling out radiative effects, has been presented by the ZEUS Collaboration together with the SM expectation. We have checked that the latter, obtained using hadronization procedures incorporated in the standard HERACLES and ARIADNE program packages, are in excellent agreement (within a few per cent) with our parton-level analysis. Any small differences which persist can be removed by calibrating the cross-section binwise, so as to yield the actual ZEUS expectations. This procedure has the added merit of taking care of residual higher order effects such as initial state radiation, which should be roughly the same in the SM as in the case when the graviton exchanges are included.

In Fig. 1, we present a graph showing the variation in the  $Q_{DA}^2$  distribution with the mass scale  $m_0$  in the RS model. For this graph, we have plotted the ratio

$$R(m_0, c_0) \equiv \frac{d\sigma_{RS}/dQ_{DA}^2}{d\sigma_{SM}/dQ_{DA}^2} \quad (12)$$

of the cross-section predicted in the RS model with the prediction of the SM, for  $c_0 = 0.1$  and  $m_0 = 80, 90$  and 100 GeV, together with the ZEUS data. It may be seen that the cross-section in the RS model (like the ADD model [12]) exhibits large deviations in the highest  $Q^2$  bins. This, of course, rapidly

approaches the SM (dotted line) if  $c_0$  is chosen smaller. Interestingly, the RS model predictions seem to show a slight diminution for intermediate values of  $Q_{DA}^2$ , which are intriguingly like the trend shown by the data. However, the experimental errors are too large to enable us to attach any significance to this circumstance. Accordingly, we take the conservative viewpoint that the data fit the SM very well and can be used to constrain new physics.



**Figure 1.** Illustrating the ratio  $R(m_0, c_0)$  of the  $Q_{DA}^2$  distribution in the RS model to that in the SM (see Eq. 12), for  $c_0 = 0.1$  and  $m_0 = 80, 90$  and  $100$  GeV. The dotted line corresponds to the SM. The ZEUS data are also shown.

Once the above simulation is set-up, we estimate the binwise cross-section

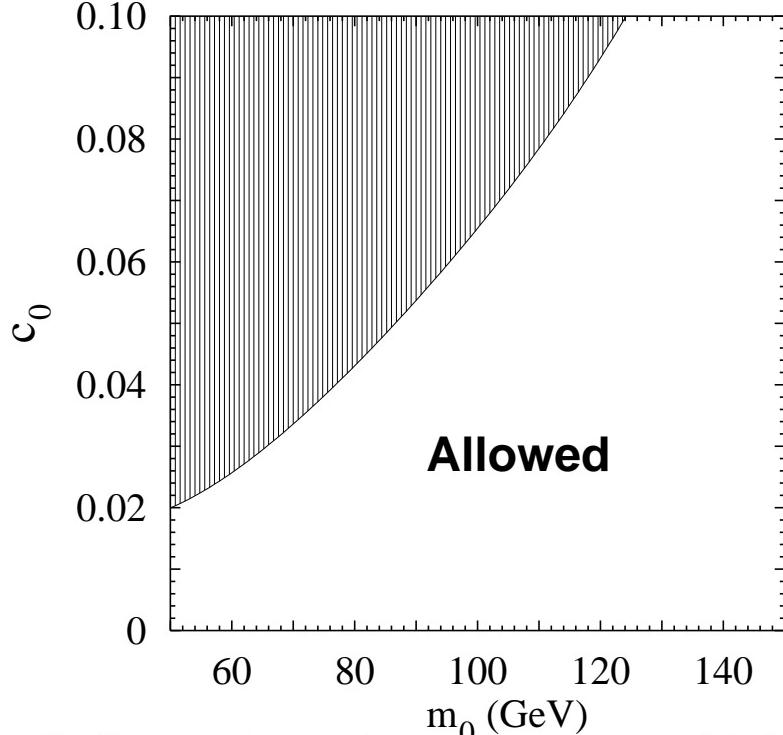
$\chi^2(m_0, c_0)$  for each value of  $m_0$  and  $c_0$  and use this distribution to calculate

$$\chi^2(m_0, c_0) = \sum_{i=1}^{20} \frac{[\sigma_i(m_0, c_0) - \sigma_i^{(C.V.)}]^2}{\epsilon_i^2} \quad (13)$$

where

$$\epsilon_i = \epsilon_1^i \theta[\sigma_i(m_0, c_0) - \sigma_i^{(C.V.)}] + \epsilon_2^i \theta[\sigma_i^{(C.V.)} - \sigma_i(m_0, c_0)] \quad (14)$$

assuming that the experimental value in the  $i$ -th bin is given by  $[\sigma_i^{(C.V.)}]_{-\epsilon_2^i}^{+\epsilon_1^i}$ . In this, it is assumed that  $\epsilon_1^i$  and  $\epsilon_2^i$  contain the statistical and systematic errors added in quadrature. The 95% C.L. bound is then obtained by requiring  $\chi^2(m_0, c_0) < 31.41$ , which is the expectation [15] from random fluctuations.



**Figure 2.** Illustrating the constraint on the parameter space of the RS model arising from an analysis of ZEUS high- $Q^2$  data. The shaded region is ruled out at the 95% C.L. level.

In Figure 2, we show the 95% C.L. constraints on the  $m_0$ - $c_0$  plane using the above technique. Since the effective graviton coupling is quadratic in  $c_0$  we expect the cross-section to rise as  $c_0$  increases — this is reflected in the fact that Figure 2 shows upper bounds on  $c_0$ . On the other hand, the  $m_0$  dependence of the cross-section is very complicated, because of the summation over the KK states. However, as the figure makes clear, there is a sharp drop in the cross-section as  $m_0$  increases, so that the ZEUS data become insensitive to the new physics beyond about  $m_0 = 120$  GeV. This corresponds to  $M_1 \simeq 460$  GeV, a value which is still not accessible to the generation of colliders running at present.

As the above figure and discussion makes clear, HERA data as presented by the ZEUS Collaboration provide somewhat modest, but nevertheless interesting constraints on the parameter space of the RS model of quantum gravity. Since gravitons couple to the energy-momentum tensor of the matter fields, one may expect considerable improvements in these results at machines running at higher energies, such as the LHC, the proposed NLC and possible muon colliders. In particular, it would be interesting to see if these machines could actually find graviton resonances, which one expects in the RS model [8], but not in the ADD theory. We have performed a preliminary study of the RS model in the light of HERA data, and we expect that the future will see many more detailed studies of this very interesting scenario.

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